



MARKSCHEME

May 2013

MATHEMATICS STATISTICS AND PROBABILITY

Higher Level

Paper 3

11 pages

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Instructions to Examiners

Abbreviations

- M** Marks awarded for attempting to use a correct **Method**; working must be seen.
- (M)** Marks awarded for **Method**; may be implied by **correct** subsequent working.
- A** Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding **M** marks.
- (A)** Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.
- R** Marks awarded for clear **Reasoning**.
- N** Marks awarded for **correct** answers if **no** working shown.
- AG** Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document “**Mathematics HL: Guidance for e-marking May 2013**”. It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the “must be seen” marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp **A0** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is not possible to award **M0** followed by **AI**, as **A** mark(s) depend on the preceding **M** mark(s), if any.
- Where **M** and **A** marks are noted on the same line, *eg* **MIAI**, this usually means **MI** for an **attempt** to use an appropriate method (*eg* substitution into a formula) and **AI** for using the **correct** values.
- Where the markscheme specifies (**M2**), **N3**, *etc.*, do **not** split the marks.
- Once a correct answer to a question or part-question is seen, ignore further working.

3 **N marks**

Award **N** marks for **correct** answers where there is **no** working.

- Do **not** award a mixture of **N** and other marks.
- There may be fewer **N** marks available than the total of **M**, **A** and **R** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.

4 **Implied marks**

Implied marks appear in **brackets eg (M1)**, and can only be awarded if **correct** work is seen or if implied in subsequent working.

- Normally the correct work is seen or implied in the next line.
- Marks **without** brackets can only be awarded for work that is **seen**.

5 **Follow through marks**

Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s). To award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part.

- If the question becomes much simpler because of an error then use discretion to award fewer **FT** marks.
- If the error leads to an inappropriate value (eg $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).
- Within a question part, once an error is made, no further **dependent A** marks can be awarded, but **M** marks may be awarded if appropriate.
- Exceptions to this rule will be explicitly noted on the markscheme.

6 **Mis-read**

If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). A candidate should be penalized only once for a particular mis-read. Use the **MR** stamp to indicate that this has been a mis-read. Then deduct the first of the marks to be awarded, even if this is an **M** mark, but award all others so that the candidate only loses one mark.

- If the question becomes much simpler because of the **MR**, then use discretion to award fewer marks.
- If the **MR** leads to an inappropriate value (eg $\sin\theta = 1.5$), do not award the mark(s) for the final answer(s).

7 **Discretionary marks (d)**

An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. In such cases the annotation **DM** should be used and a brief **note** written next to the mark explaining this decision.

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, etc.
- Alternative solutions for part-questions are indicated by **EITHER . . . OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

Unless the question specifies otherwise, **accept** equivalent forms.

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

Example: for differentiating $f(x) = 2 \sin(5x - 3)$, the markscheme gives:

$$f'(x) = (2 \cos(5x - 3))5 \quad (=10 \cos(5x - 3)) \quad \text{AI}$$

Award **AI** for $(2 \cos(5x - 3))5$, even if $10 \cos(5x - 3)$ is not seen.

10 Accuracy of Answers

Candidates should **NO LONGER** be penalized for an accuracy error (**AP**).

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy. When this is not specified in the question, all numerical answers should be given exactly or correct to three significant figures. Please check work carefully for **FT**.

11 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

12 Calculators

A GDC is required for paper 3, but calculators with symbolic manipulation features (eg TI-89) are not allowed.

Calculator notation

The Mathematics HL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

13 More than one solution

Where a candidate offers two or more different answers to the same question, an examiner should only mark the first response unless the candidate indicates otherwise.

1. (a) $\bar{x} = 14$ AI
 $s_{n-1}^2 = \frac{3977.57}{19} - \frac{280^2}{380}$ (MI)
 $= 3.03$ AI

[3 marks]

Note: Accept any notation for these estimates including μ and σ^2 .

Note: Award **MOA0** for division by 20.

- (b) the 95% confidence limits are (MI)
 $\bar{x} \pm t \sqrt{\frac{s_{n-1}^2}{n}}$

Note: Award **M0** for use of z .

ie, $14 \pm 2.093 \sqrt{\frac{3.03}{20}}$ (AI)

Note: **FT** their mean and variance from (a).

giving [13.2, 14.8] AI

Note: Accept any answers which round to 13.2 and 14.8.

[3 marks]

- (c) Use of t-statistic $\left(= \frac{14 - 15}{\sqrt{\frac{3.03}{20}}} \right)$ (MI)

Note: **FT** their mean and variance from (a).

Note: Award **M0** for use of z .

Note: Accept $\frac{15 - 14}{\sqrt{\frac{3.03}{20}}}$.

$= -2.569...$ (AI)

Note: Accept 2.569...

$p\text{-value} = 0.009392... \times 2 = 0.0188$ AI

Note: Accept any answer that rounds to 0.019.

Note: Award **(MI)(AI)A0** for any answer that rounds to 0.0094.

insufficient evidence to reject H_0 (or equivalent, eg accept H_0 or reject H_1) RI

Note: **FT** on their p -value.

[4 marks]

Total [10 marks]

2. (a) H_0 : number of goals can be modelled by a Poisson distribution
 H_1 : number of goals cannot be modelled by a Poisson distribution **A1**

Note: Do not award **A1** if a value for the mean of the distribution is given.

[1 mark]

- (b) (i) sample mean = $\frac{\sum fx}{\sum f}$ **(M1)**
 $= 2.3$ **A1**
 exp freq for r goals = $60 \times \frac{e^{-2.3} \times 2.3^r}{r!}$ ($r \leq 4$) **(M1)**

| | | | | | | |
|--------------------|--------|---------|---------|---------|--------|----------|
| Number of goals | 0 | 1 | 2 | 3 | 4 | ≥ 5 |
| Expected Frequency | 6.0155 | 13.8357 | 15.9111 | 12.1985 | 7.0141 | 5.0250 |

A3

Note: At this stage, accept tabular values correct to 3 significant figures.

Note: Award **A2** for 1 error, **A1** for 2 errors and **A0** for 3 or more errors.

- (ii) $\chi^2_{calc} = \sum \frac{f_o^2}{f_e} - N$ or $\sum \frac{(f_o - f_e)^2}{f_e} = 2.69$ **(M1)A1**

Note: Do not **FT** from incorrect tabular values.

Note: Accept any answer that rounds to 2.7.

DF = 4 **(A1)**

Note: **FT** the DF from the table, *ie* award the **(A1)** if the value given is 2 less than the number of cells.

p -value = 0.612 **A1**

Note: Accept any answer that rounds to 0.61.

Note: Do not **FT** from incorrect tabular values.

- (iii) the manager's belief is supported (at all reasonable significance levels) **R1**
 (or equivalent, *eg* accept H_0 or reject H_1)

Note: Follow through their p -value or χ^2_{crit} .

[11 marks]

Total [12 marks]

3. (a) $H_0 : \mu = 1.2; H_1 : \mu < 1.2$ AI

Note: Accept “ $H_0 : (30\text{-day}) \text{ mean} = 36; H_1 : (30\text{-day}) \text{ mean} < 36$ ”.

[1 mark]

- (b) (i) let X denote the number of breakdowns in 30 days
 then under $H_0, E(X) = 36$ (AI)
 sig level = $P(X \leq 25 | \text{mean} = 36)$ (MI)(AI)
 = 0.0345 (3.45%) AI

Note: Accept any answer that rounds to 0.035 (3.5%).

Note: Do not accept the use of a normal approximation.

- (ii) under $H_1, E(X) = 22.5$ (AI)
 $P(\text{Type II error}) = P(X \geq 26 | \text{mean} = 22.5)$ (MI)(AI)
 = 0.257 AI

Note: Accept any answer that rounds to 0.26.

Note: Do not accept the use of a normal approximation.

[8 marks]

Total [9 marks]

4. (a) (i) $F(x) = \int_1^x \frac{3u^2 + 2u}{10} du$ (MI)
 $= \left[\frac{u^3 + u^2}{10} \right]_1^x$ AI

Note: Do not penalise missing or wrong limits at this stage.
 Accept the use of x in the integrand.

$$= \frac{x^3 + x^2 - 2}{10} \quad \text{AI}$$

- (ii) the median m satisfies the equation $F(m) = \frac{1}{2}$ so (MI)
 $m^3 + m^2 - 7 = 0$ (AI)

Note: Do not **FT** from an incorrect $F(x)$.

$$m = 1.63 \quad \text{AI}$$

Note: Accept any answer that rounds to 1.6.

[6 marks]

continued ...

Question 4 continued

- (b) (i) the mean of a large sample from any distribution is approximately normal

AI

Note: This is the minimum acceptable explanation.

- (ii) we require the mean μ and variance σ^2 of X

$$\mu = \int_1^2 \left(\frac{3x^3 + 2x^2}{10} \right) dx \quad (M1)$$

$$= \frac{191}{120} (1.591666\dots) \quad AI$$

$$\sigma^2 = \int_1^2 \left(\frac{3x^4 + 2x^3}{10} \right) dx - \mu^2 \quad (M1)$$

$$= 0.07659722\dots \quad AI$$

the central limit theorem states that

$$\bar{X} \approx N \left(\mu, \frac{\sigma^2}{n} \right), \text{ i.e. } N(1.591666\dots, 0.0005106481\dots) \quad M1A1$$

$$P(\bar{X} > 1.6) = 0.356 \quad AI$$

Note: Accept any answer that rounds to 0.36.

[8 marks]

Total [14 marks]

5. (a) (i) the number of hits, $X \sim B(8, 0.4)$ (A1)

$$P(X = 4) = \binom{8}{4} \times 0.4^4 \times 0.6^4$$
 (M1)

$$= 0.232$$
 A1

Note: Accept any answer that rounds to 0.23.

- (ii) let the 4th hit occur on the Y^{th} shot so that $Y \sim NB(4, 0.4)$ (A1)

$$P(Y = 8) = \binom{7}{3} \times 0.4^4 \times 0.6^4$$
 (M1)

$$= 0.116$$
 A1

Note: Accept any answer that rounds to 0.12.

[6 marks]

- (b) (i) $X \sim NB(10, 0.4)$ (M1)

$$E(X) = \frac{10}{0.4} = 25$$
 A1

- (ii) let P_x denote $P(X = x)$

$$P_x = \binom{x-1}{9} \times 0.4^{10} \times 0.6^{x-10}$$
 A1

$$\frac{P_x}{P_{x-1}} = \frac{\binom{x-1}{9} \times 0.4^{10} \times 0.6^{x-10}}{\binom{x-2}{9} \times 0.4^{10} \times 0.6^{x-11}}$$
 M1A1

$$= \frac{(x-1)!}{9!(x-10)!} \times \frac{9!(x-11)! \times 0.6}{(x-2)!}$$
 A1

Note: Award A1 for correct evaluation of combinatorial terms.

$$= \frac{3(x-1)}{5(x-10)}$$
 AG

- (iii) $P_x > P_{x-1}$ as long as
 $3x - 3 > 5x - 50$ (M1)
i.e. $x < 23.5$ (A1)
 the most likely value is 23 A1

Note: Allow solutions based on creating a table of values of P_x .

[9 marks]

Total [15 marks]